دوال يجب حفظها

$\boxed{1}$ $A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$

$\boxed{2}$ $e^{-t}\cdot u(t) \rightleftharpoons \frac{1}{1+J2\pi f}$ as $\boxed{3}$ but $\alpha=1$

$\boxed{3}$ $e^{-\alpha t}\cdot u(t) \rightleftharpoons \frac{1}{\alpha+J2\pi f}$

$\boxed{4}$ $e^{\alpha t}\cdot u(-t) \rightleftharpoons \frac{1}{\alpha-J2\pi f}$

$\boxed{5}$ $e^{t}\cdot u(-t) \rightleftharpoons \frac{1}{1-J2\pi f}$ as $\boxed{4}$ but $\alpha - 1$

$\boxed{6}$ $sgn(t) \rightleftharpoons \frac{1}{J\pi f}$

$\boxed{7}$ $u(t) \rightleftharpoons \frac{1}{2}\left[\frac{1}{J\pi f}+\delta(f)\right]$

$\boxed{8}$ $A \rightleftharpoons A\cdot\delta(f)$

$\boxed{9}$ $\delta(t) \rightleftharpoons 1$

$\boxed{10}$ $m(t)\cdot\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[M(f-f_c)+M(f+f_c)\right]$ freq. shift

$\boxed{11}$ $A_c\cdot\cos(2\pi f_c t) \rightleftharpoons \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]$ freq. shift

$\boxed{12}$ $A\tau\,\text{Sinc}(t\tau) \rightleftharpoons A\,\text{rect}(f/\tau)$ duality with $\boxed{1}$

$\boxed{13}$ $1\cdot e^{-J2\pi f_0 t} \rightleftharpoons \delta(f+f_0)$ freq. shift

Page
Date

When you're down and troubled
And you need some love care

إثباتات الخواص

$$* \; G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \, dt \qquad \text{F.T.}$$

$$* \; g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \, df \qquad \text{I.F.T.}$$

① Linearity

$$\int_{-\infty}^{\infty} \left[a \cdot g_1(t) + b \cdot g_2(t) \right] e^{-j2\pi f t} \, dt$$

$$= \int_{-\infty}^{\infty} a \cdot g_1(t) \, e^{-j2\pi f t} \, dt + \underbrace{b \int_{-\infty}^{\infty} g_2(t) \cdot e^{-j2\pi f t} \, dt}_{\text{معادلة F.T. لـ } g_2(t)}$$

$$= a \cdot G_1(f) + b \cdot G_2(f)$$

② Time Scaling

تتحول $t \to \tau$ بدل $dt \to d\tau$

$$F[g(at)] = \int_{-\infty}^{\infty} g(at) \cdot e^{-j2\pi f t} \, dt \longrightarrow \boxed{1}$$

$$\text{let } \tau = at \longrightarrow d\tau = a \cdot dt$$

$$t = \tau/a \qquad dt = \frac{d\tau}{a}$$

عوض في $\boxed{1}$

$$F[g(at)] = \frac{1}{a} \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi \frac{f}{a} \tau} \cdot d\tau$$

$$= \frac{1}{a} \cdot G(f/a)$$

$\boxed{1}$

③ Duality

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \, df$$

replace t by $-t$ in both sides

$$g(-t) = \int_{-\infty}^{\infty} G(f) \cdot e^{-j2\pi f t} \, df$$

replace t by f and f by t

$$\therefore g(-f) = \int_{-\infty}^{\infty} G(t) \cdot e^{-j2\pi f t} \, dt$$

$$\therefore G(t) \rightleftharpoons g(-f)$$

④ Time Shift

t-to حطها

$$F[g(t-t_0)] = \int_{-\infty}^{\infty} g(t-t_0) \cdot e^{-j2\pi f t} \, dt$$

حطها t-to

let $\tau = t - t_0 \rightarrow d\tau = dt$

$t = \tau + t_0$

$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f(\tau + t_0)} \, d\tau$$

$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f \tau} \cdot \overbrace{e^{-j2\pi f t_0}} \, d\tau$$

ثابت يطلع
برا التكامل.

$$= e^{-j2\pi f t_0} \cdot \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f \tau} \, d\tau$$

$$= e^{-j2\pi f t_0} \cdot G(f)$$

⑤ Frequency Shift

$$F[g(t)\cdot e^{-j2\pi f_o t}] = \int_{-\infty}^{\infty} g(t)\cdot e^{-j2\pi f_o t}\cdot e^{-j2\pi f t}\,dt$$

$$= \int_{-\infty}^{\infty} g(t)\cdot e^{-j2\pi(f+f_o)t}\,dt$$

$$= G(f+f_o)$$

⑥ Area under g(t)

$$\text{Area} = \int_{-\infty}^{\infty} g(t)\cdot dt = \text{fourier Transform eq. at } f=0$$

$$= \int_{-\infty}^{\infty} g(t)\cdot \underbrace{e^{-j2\pi f t}}_{\text{at } f=0}\,dt$$

$$= G(0)$$

⑦ Area under G(f)

$$\text{Area} = \int_{-\infty}^{\infty} G(f)\,df = \text{inverse fourier eq. at } t=0$$

$$= \int_{-\infty}^{\infty} G(f)\cdot \underbrace{e^{+j2\pi f t}}_{\text{at } t=0}\,df$$

$$= g(0)$$

⑧ Differentiation in time domain

$$\because\ g(t) = \int_{-\infty}^{\infty} G(f)\cdot e^{+j2\pi f t}\,df$$

and $g(t) \rightleftharpoons G(f)$

$$\frac{d}{dt} g(t) = \int_{-\infty}^{\infty} (j2\pi f) . G(f) . e^{+j2\pi f t} \, df$$

$$\therefore \frac{d}{dt} g(t) \rightleftharpoons (j2\pi f) . G(f)$$

⑨ Integeration in time

$$\& \quad \frac{d^n}{dt} g(t) \rightleftharpoons (j2\pi f)^n . G(f)$$

Prove : $\int_{-\infty}^{\infty} g(t) \, dt \xrightarrow{F.T.} \frac{1}{j2\pi f} G(f)$

$$\because \quad g(t) = \frac{d}{dt} \left[\int_{-\infty}^{\infty} g(t) \, dt \right]$$

$$\downarrow F.T.$$

from $\boxed{8}$

$$G(f) = (j2\pi f) . F\left[\int_{-\infty}^{\infty} g(t) \, dt \right]$$

$$\therefore F\left[\int_{-\infty}^{\infty} g(t) \, dt \right] = \frac{G(f)}{(j2\pi f)}$$